Acceleration and ejection of ring vortexes by a convergent flow as a probable mechanism of arising jet components of AGN

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Abstract

Exact solutions of two-dimensional hydrodynamics equations for the symmetric configurations of two and four vortices in the presence of an arbitrary flow with a singular point are found. The solutions describe the dynamics of the dipole toroidal vortex in accretion and wind flows in active galactic nuclei. It is shown that the toroidal vortices in a converging (accretion) flow, being compressed along the large radius, are ejected with acceleration along the axis of symmetry of the nucleus, forming the components of two-sided jet. The increment of velocities of the vortices is determined by the monopole component of the flow only. The dipole component of the flow determines the asymmetry of ejections in the case of an asymmetric flow.

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1. **Introduction**

A large number of works (see, for example, the monograph [1]) is devoted to the origin of jets. In the most of them the decisive role is played by a strong magnetic field [2-4], or "external", or arising from the development of instabilities in the plasma of the accretion disk. This field serves as a guide for the movement of particles under the action of electromagnetic, centrifugal and gravitational forces, allowing them to move against the gravity and carry away the angular momentum that is necessary for the effective accretion process, which is responsible for the activity of the nucleus (see discussion and references in reviews [5-7]). At the same time, the very possibility of the existence of strong magnetic fields in the accretion disks around the black holes is not entirely clear. In this connection, the models of continuous flow without magnetic field are also considered, including those, in which the structure of jets resemble the hydrodynamic tornadoes [8].

The observations, however, show that at small distances from the nucleus (parsec scales for active galactic nuclei (AGN)) the emergence of the individual (including the superluminal) components of the radio jets are observed [9]. In the model we discuss here [10-12] some ejections are derived from the kinematics of the interaction of vortices and the exposure of the magnetic field is not required. An important role in this process has the flow that can affect the velocity of ejection. Therewith, greater velocities of the ejected components are attained in a converging (accretion) flow.

We regard a system of toroidal vortices [10], surrounding the central part of AGN, the outermost of which is observed as a "obscuring torus". The vortex motion arises in the torus due to twisting by the wind and radiation. Due to the flow symmetry the movement in the torus possesses the dipole character (Fig. 1 in [10]). In its simplest form, this movement can be represented as the motion of two opposite rotating vortex rings in a radial flow. As is known, the dynamics of a vortex ring can be described as the movement of a pair of point vortices that arise in the cross-section of the ring (torus) by the plane of symmetry. In our case it is a symmetrical system of two or four vortices (or two vortex pairs) in an arbitrary flow with a point singularity.

2. **Setting of a problem**

We consider the motion of a system of point vortices on a background flow caused by the stationary singular point, which is placed at the origin. Following [13, 14], the stream function can be represented as the sum of two constituents: its regular part ψ_{reg} which describes the background flow and the singular part ψ_{sing}, which describes the point vortices.

The complex potential of the flow can be represented as a series

$$w_{reg} = C_0 \ln z + \frac{C_1}{z} + \frac{C_2}{z^2} + ... \qquad (z = x + iy) \tag{1}$$

When the flow is symmetric about the axis Ox, all the coefficients C_k in (1) are real. Expressing the complex potential through the usual velocity potential φ and the stream function ψ according to $w = \varphi + i\psi$ and passing on to the polar coordinates $z = r \cdot \exp{(i\theta)}$ we obtain the stream function in the form

$$\psi_{reg} = C_0\theta - \frac{C_1}{r}\sin\theta - \frac{C_2}{r^2}\sin 2\theta - .... \tag{2}$$

The radial and azimuthal velocity components of the background flow are determined by the conditions

$$v_r = \frac{1}{r}\frac{\partial\psi_{reg}}{\partial\theta}, \quad v_\theta = -\frac{\partial\psi_{reg}}{\partial r} \ .$$

Combining the monopole at the origin (source or sink), dipole, quadrupole, ... with intensity C_0, C_1, C_2, ... we can obtain a given distribution of the radial velocity component at the circle $\mid z \mid= R$ which describes the considered flow. The system of vortices and the flow has Hamiltonian form.

3. **Symmetric motion of two vortex pairs in the flow with singularity of the type "source + quadrupole + ..."**

Now we consider the dynamics of a system of two vortex pairs in the stream flow generated by a fixed singular point provided the existence of two axes of symmetry (fig.1). This case can be interpreted as the motion of a point vortex in the right angle (which sides are impermeable "walls") at the apex of which is placed the referred hydrodynamic singularity. Note that

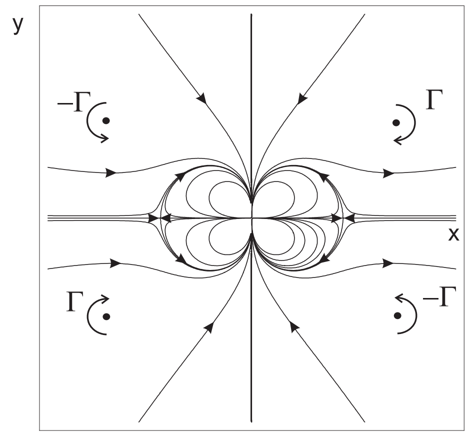

Figure 1: *Scheme of the movement of four vortices (two vortex pairs) in a symmetric flow, containing the sink and the quadrupole for* $C_0 = -1$, $C_1 = 0$, $C_2 = 1$, $\Gamma = -4\pi$

the solution of the problem for a symmetric system of four vortices in the absence of the background flow was found in the classical Grobli's work (see [15, 16]). In a purely radial flow it admits a Hamiltonian formulation [11] and the exact solution of the dynamic problem [11, 12].

The stream function for the discussed flow is represented in the form

$$\psi_{reg} = C_0\theta - \frac{C_2}{r^2}\sin 2\theta - \frac{C_4}{r^4}\sin 4\theta - .... \tag{3}$$

The vortex components of the right pair are arranged symmetrically about the axis Ox in the points ($x;\ y$) and (x;-y), and the intrinsic (due to the interaction of vortices and not related to the presence of the background flow) velocity of the vortex in the 1-st quadrant is

$$\overrightarrow{V}_{sing} = \frac{\Gamma}{4\pi}\left\{\frac{1}{y} - \frac{y}{x^2+y^2};\ \frac{x}{x^2+y^2} - \frac{1}{x}\right\}.$$

In the case of only one vortex pair without a background flow that corresponds to the known expression

$$\overrightarrow{V}_s = (\Gamma/4\pi y; 0)\,, \tag{4}$$

where Γ is the intensity of the vortex, x and y are its abscissa and ordinate.

The dynamics of two pairs of vortices in the flow is described by the equations

$$\dot{x} = \frac{\Gamma}{4\pi}\left(\frac{1}{y} - \frac{y}{x^2+y^2}\right) + \frac{\partial\psi_{reg}}{\partial y};\ \dot{y} = \left(\frac{x}{x^2+y^2} - \frac{1}{x}\right) - \frac{\partial\psi_{reg}}{\partial x} \tag{5}$$

and the Hamiltonian of the system of 4 vortices is reduced to the form

$$H = \frac{\Gamma}{4\pi}\ln\frac{xy}{\sqrt{x^2+y^2}} + C_0 \arctan\frac{y}{x} - C_2\frac{2xy}{\left(x^2+y^2\right)^2} - .... \tag{6}$$

Accordingly, the equations (5) can be represented as

$$\dot{x} = \frac{\partial H}{\partial y}, \quad \dot{y} = -\frac{\partial H}{\partial x}$$

Equation $H = E = const$ determines the trajectory of vortices. Moreover, if the motion is unbounded, then the vortex pair comes from infinity along one axis (Ox), exchange their components and goes away to infinity along the other axis (Oy):

$$y_\infty = \exp\left\{\frac{4\pi E}{\Gamma}\right\},\ x \to +\infty; \quad x_\infty = \exp\left\{\frac{4\pi(E - C_0\frac{\pi}{2})}{\Gamma}\right\},\ y \to +\infty;$$

$$C_0 \neq 0,\ C_1 \neq 0,\ C_2 \neq 0.$$

For the asymptotic values x_∞ and y_∞ coordinates of the vortex moving in the first quadrant, we have the correlation

$$y_\infty = x_\infty exp\left(2\pi^2 C_0/\Gamma\right), \tag{7}$$

which coincides with the result for the purely radial flow [11,12]. This means that the ratio y_∞/x_∞ of the limit distances between elements of the vortex pairs at infinity and, correspondingly, the ratio of their velocities, are only determined by the intensity of the C_0 of the source (sink) at the origin and do not depend on the other multipole components.

Since the velocity of the translational motion of a vortex pair is inversely proportional to the distance between them (cf. (4)), then we obtain the ratio of the asymptotic values of the velocities of the pairs, coming from infinity $V_- = \Gamma/(4\pi y_\infty)$ and going to infinity $V_+ = \Gamma/(4\pi x_\infty)$

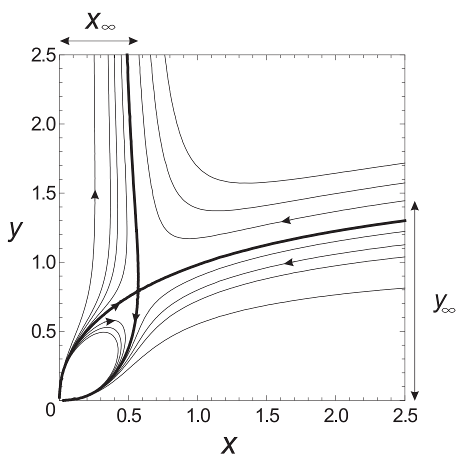

Figure 2: *Phase portrait for the vortex of the first quadrant, corresponding to the movement of the four vortices in the accretion flow.* $C_0 = -1, \quad C_1 = 0, C_2 = 1, \quad \Gamma = -4\pi$. *Thin and bold lines correspond to the trajectories of the vortex and the separatrix accordingly. The asymptotic values of coordinates (x_∞ and y_∞) correspond to half of the distance between the components of the pairs of vortices at the inlet and outlet of the system.*

$$V_-/V_+ = x_\infty/y_\infty = exp\left(-2\pi^2 C_0/\Gamma\right), \tag{8}$$

that coincides with the ratio obtained in [11, 12]. Obviously, the vortex pairs go away from the source at a slower velocity, and from the sink – corresponding to accretion – with a greater velocity than they come in. Thus, the obtained solutions confirm the main result of the work [12] about the acceleration of ejections by the radial accretion flow, and expand it on the general case of a symmetric accretion flow. As we will show below the dipole component of the flow may be responsible for the asymmetry of ejections.

4. Motion of two vortex pairs in the flow from dipole which axis is the axis of symmetry of the flow

In the case of motion of two vortex pairs in the flow from a dipole with the axis Oy with intensity C_1 (that corresponds to replacement $C_1 \to iC_1$ in (1)) the equations of dynamics of the vortices can be expressed through the

coordinates of the two vortices in the right half-plane in the form

$$\dot{x}_1 = \frac{\partial H}{\partial y_1} \quad \dot{y}_1 = -\frac{\partial H}{\partial x_1} \quad \dot{x}_2 = -\frac{\partial H}{\partial y_2} \quad \dot{y}_2 = \frac{\partial H}{\partial x_2};$$

$$H = \frac{C_1 x_1}{x_1^2 + y_1^2} - \frac{C_1 x_2}{x_2^2 + y_2^2} + \frac{\Gamma}{4\pi} \ln\left((x_1 - x_2)^2 + (y_1 - y_2)^2\right) - \tag{9}$$

$$\frac{\Gamma}{4\pi} \ln\left((x_1 + x_2)^2 + (y_1 - y_2)^2\right) + \frac{\Gamma}{4\pi} \ln\, x_1 + \frac{\Gamma}{4\pi} \ln\, x_2.$$

The vortex moving in the first quadrant has the intensity Γ and coordinates $(x_1; y_1)$ and the vortex in the fourth quadrant has the intensity -Γ and coordinates $(x_2; y_2)$. The vortices located in the left half-plane have the parameters -$\Gamma, (-x_1; y_1)$ and $\Gamma, (-x_2; y_2)$ accordingly.

From the first integral $H = E = const$ one can get a relationship between the asymptotic values $y_\infty, x_{1\infty}, x_{2\infty}$: the half of the distance (y_∞) between the elements of the vortex pairs coming from infinity along the axis Ox, and the same for the pairs going to infinity along the axis Oy ($x_{1\infty}$ and $x_{2\infty}$)

$$x_{1\infty} x_{2\infty} = y_\infty^2. \tag{10}$$

The dipole power occurs implicitly through the difference of asymptotics $x_{1\infty}$ and $x_{2\infty}$. Really, as it is easily seen from the fig.3, the interaction between the vortices gives the same distortion in the x direction for both the 1-st and 2-nd vortices. The difference only arises due to the influence of the flow.

In the case when there is also a source (sink) with intensity C_0 in the origin, the relationship (10) becomes

$$x_{1\infty} x_{2\infty} = y_\infty^2 exp \left\{ -\frac{4\pi^2 C_0}{\Gamma} \right\}. \tag{11}$$

Note that the relation (11) remains valid if there are also the multipoles of the higher orders. It has been taken into account that the initial asymptotics corresponds to the symmetric pair of vortices. In terms of the ring vortices this corresponds to the dipole toroidal vortex of "infinite" radius, which is compressed by the interaction of the ring components. Dipole flow introduces an asymmetry in the motion, and different velocity correspond to vortex rings of different radius thrown in opposite directions. Thus, within the framework of the dipole-toroidal model one can naturally explain both the very appearance of the ejections accelerated by the accretion flow and the observed asymmetry of the ejections in AGNs.

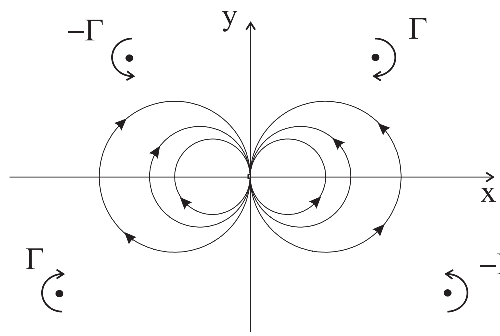

Figure 3: *The scheme of motion of four vortices in the dipole flow which destroys the symmetry relatively of the axis Ox. It is shown the asymmetry of the movement of vortices in such a stream.*

Conclusion

The model of active galactic nuclei proposed in [10] gives the possibility within the framework of hydrodynamics of an ideal incompressible fluid to study the dynamic behavior of toroidal structures and the influence on their movement of accretion-wind flows.

In this paper we considered a simplified plane model, for which the analogue of a vortex ring is a pair of point vortices, which axis coincides with the axis of the ring. The dipole-vortex structure of the torus in the 2D model is represented by the two pairs of vortices with a common axes and the angular momenta of the opposite signs. It takes into account that the symmetric pairs of vortices correspond to the initial asymptotics.

In the terms of the ring vortices that corresponds to initial dipole toroidal vortex of "infinite" radius which is compressed due to interaction of the ring components. In the absence of the background flow this problem resembles the classical problem of the Helmholtz vortex ring interaction with the wall parallel to the plane in which the vortex ring lies. The wall can be replaced by a mirror image of the vortex, and the problem can be reduced to the interaction of oppositely rotating vortex rings. However, in our case the direction of rotation is opposite to the corresponding to approaching the vortex to the wall, which was considered by Helmholtz. (Our choice corresponds to the moving off the vortex from the wall.) In a purely radial flow it admits a Hamiltonian formulation and exact solution of the dynamic problem [11, 12].

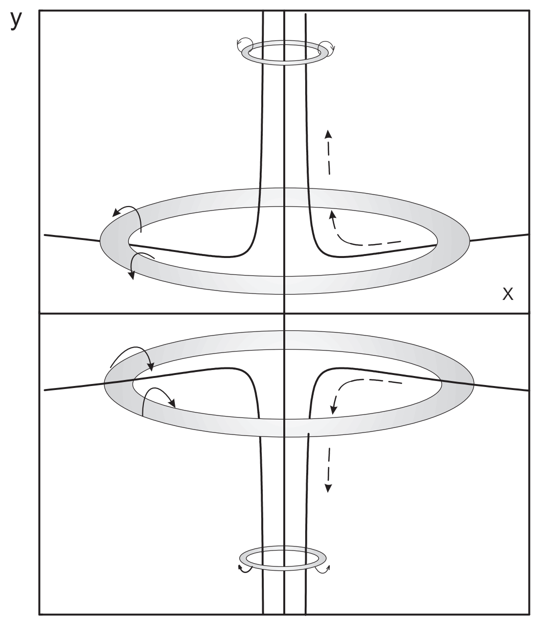

Figure 4: *Scheme of the motion of ring vortices in a converging flow, appropriate to the discussed planar analog (symmetrical flow [12]).*

In this study the conclusion about acceleration of the ejections by the radial accretion flow is expanded on the general case of two-dimensional flow. It is shown that the monopole component of the flow is only responsible for the acceleration of ejections, and a dipole flow component may be responsible for the asymmetry of bilateral ejections.

Thus, the dipole-toroidal model can naturally explain both the very emergence of ejections, accelerated by the accretion flow in the case of the general character of the flow, and the observed asymmetry of the ejections of active galactic nuclei and quasars.

The text basically corresponds to [17]. We have added some figures and corrected typos in formulas.

For an extended discussion of this subject see [18].

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References

1. V.S. Beskin. Axisymmetric steady flows in astrophysics, M.: Fizmatlit, 2006, in Russian; MHD Flows in Compact Astrophysical Objects, Springer, 2010, 425 pp.
2. G. Bisnovatyi-Kogan & A. Ruzmaikin, Ap & Space Sci, 42, 401 (1976).
3. R.D. Blandford, MNRAS, 176, 465 (1976).
4. R.V.E. Lovelace, Nature, 262, 649 (1976).
5. D. Lynden-Bell, Mon. Not. R. Astron. Soc. 369, 1167 (2006).
6. R.D. Blandford, Phil.Trans.R.Soc.Lond. A, 358, 811 (2000).
7. I.F. Mirabel, Phil.Trans.R.Soc.Lond. A, 358, 841 (2000).
8. M.G. Abrahamian, Astrofizika, 51, 201, 431, 617 (2008).
9. G.V. Vermeulen & M.H. Cohen, ApJ, 430, 467 (1994).
10. E.Yu. Bannikova & V.M. Kontorovich, Astron. J., 84, 298 (2007), astro-ph/0707.1478.
11. E.Yu. Bannikova, V.M. Kontorovich & G.M. Resnick, JETP, 132, 3, 615 (2007).
12. E.Yu. Bannikova & V.M. Kontorovich, Phys.Lett. A, 373, 1856 (2009).
13. G.M. Reznik, J. Fluid. Mech. 240, 405 (1992).
14. G. Reznik & Z. Kizner, Theor. & Comp. Fluid Dynamics, 24, # 1-4, 65-75 (2010).
15. A.V. Borisov & I.S. Mamaev. Mathematical methods for the dynamics of vortex structures. Moscow-Izhevsk, ICI, 2005, 368 pp.
16. W. Grobli, Vierteljahrsch. d. Naturforsch.Geselsch., 22, 37, 129 (1887).
17. E.Yu. Bannikova, V.M. Kontorovich & C.A. Poslavsky, In: Transformation of waves, coherent structures and turbulence, M: LENAND, 2009, P.304.
18. C.A. Poslavsky, E.Yu. Bannikova & V.M. Kontorovich, Astrophysics, Vol. 53, No. 2, 174 (2010)